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Original Research Article

Impact of different centroid means on the accuracy of orthometric height modelling by geometric geoid method

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ABSTRACT

Background: Orthometric height, as well as geoid modelling using the geometric method, requires centroid computation. And this can be obtained using various models, as well as methods. These methods of centroid mean computation have impacts on the accuracy of the geoid model since the basis of the development of the theory of each centroid mean type is different. This paper presents the impact of different centroid means on the accuracy of orthometric height modelling by geometric geoid method.
Methods: DGPS observation was carried out to obtain the coordinates and ellipsoidal heights of selected points. The centroid means were computed with the coordinates using three different centroid means models (arithmetic mean, root mean square and harmonic mean). The computed centroid means were entered accordingly into a Microsoft Excel program developed using the Multiquadratic surface to obtain the model orthometric heights at various centroid means. The root means square error (RMSE) index was applied to obtain the accuracy of the model using the known and the model orthometric heights obtained at various centroid means.
Results: The computed accuracy shows that the arithmetic mean method is the best among the three centroid means types.
Conclusions: It is concluded that the arithmetic mean method should be adopted for centroid computation, as well as orthometric height modelling using the geometric method.

Keywords: Geoid, Centroid, Arithmetic, Root mean square, Harmonic, Mean, Orthometric height

INTRODUCTION

A centroid is defined as the point whose coordinates are the average values of the coordinates of all points of the figure (polygon).[1] It relates to the concept of central tendency of mean, median and mode.[2] Further, that the mean is a type of average that is a single value used to represent every element in the dataset but can be classified into arithmetic mean, root mean square, harmonic mean and geometric mean.

It is very important to realize that mean has a centring property which implies that when it (mean) is subtracted from each observation in the dataset, the sum of the differences is zero for a normally distributed data. The use of normalized coordinates in the interpolation of geoid undulation and determination of orthometric heights require of the centroid mean techniques along with the observed coordinates in the absolute form. This is required for referencing coordinates for manipulation and computation of geoid and subsequent geoid computation.

Real numbers are governed by the following features of: measurability i.e. has three sides on a number line: infinite negative side; infinite positive side; a zero in between. Value can be attached e.g. 7.345. Manipulated by mathematical operations (subtraction, addition, division and multiplication) of real numbers to yield another set of real numbers.

e.g., 5 × 4 = 20, 3 / 2 = 1.5

Due to the features of real numbers espoused above, some relationships were derived to enable the computation of different centroid means as listed under the theoretical background.[3] In summary, a commonly accepted meaning of the centroid is equivalent to the ‘centre of mass’.[4] Centroid determination allows x, y coordinates of an area to locate the points whose orthometric heights have the least root mean square error (RMSE) among arithmetic mean, root mean square mean and harmonic mean. In this, the centroid is computed with the same dataset using the arithmetic mean, root mean square mean and harmonic mean. The differences between the various centroid mean types and the observations (orthometric heights) are evaluated to obtain the residuals. The centroid mean type with the least sum of the square of the residuals has the least root mean square error. Thus, has the highest accuracy.

Practical application area of centroid principle is in the modelling of various aspects of ‘orientation disorder in crystals in electron density distribution for example in distraction experiments’ i.e., in crystallographic problems.[5] Other areas are in geoidal undulation determination and terrain correction in gravity studies.

The ellipsoid and the geoid reference surfaces are used to represent the figure of the earth that are involved in geodetic, as well as geophysical applications. Several studies have determined the local geoid models of various areas using the geometric method.[6-9] This method requires the computation of the centroid of the study area. The centroid computation can be done with several methods such as geometric mean, arithmetic mean, root mean square and harmonic mean centroids. Each of these methods of centroid computation has an effect on the accuracy of the determined geoid model as the development of each centroid mean type is based on different theories. None of the studies on local geoid model determination using the geometric method has applied the various methods of centroid mean computation and compared their accuracy to determine the best method. So, there is a need to determine the impact, as well as the effect of the various methods of centroid computation on the accuracy of the geometric geoid, as well as orthometric height modelling. The objectives of this study are: to determine and compare the impacts of different centroid means on the accuracy of orthometric height modelling by geometric geoid method and to determine the best method for centroid mean computation.

Theoretical background

Mean as a type of average comprises geometric, arithmetic, root mean square and harmonic means.[3] The four centroid means and the geometric means are given in equation.[3]

$$M_P = \left(\frac{x_1^P + x_2^P + ... + x_n^P}{n} \right)^{1/P} \qquad (1)$$

Where,

M_P is pth-power mean

$x_1, x_2, ..., x_n$ set of real numbers

Geometric mean

The geometric mean *G* is given as

$$G = (x_1, x_2, ..., x_n)^{1/n} \qquad (2)$$

Arithmetic mean centroid (M_1)

The arithmetic mean centroid is obtained by putting *p* = 1 into equation (1), the model for the computation of the centroid $(\bar{x}, \bar{y})$ mean is given in equation (3).[4]

$$\bar{x} = \frac{\sum_{k=1}^{n} x_k}{n} \text{ and } \bar{y} = \frac{\sum_{k=1}^{n} y_k}{n} \qquad (3)$$

Root mean square centroid (M_2)

Substituting *p* = 2 into equation (1), the root mean square centroid $(\bar{x}, \bar{y})$ can be computed as

$$\bar{x} = \sqrt{\frac{\sum_{k=1}^{n} x_k^2}{n}} \text{ and } \bar{y} = \sqrt{\frac{\sum_{k=1}^{n} y_k^2}{n}} \qquad (4)$$

Harmonic mean centroid (M_{-1})

Also, p = -1, the harmonic centroid $(\bar{x}, \bar{y})$ can be computed as

$$\frac{1}{\bar{x}} = \frac{\sum_{k=1}^{n} \frac{1}{x_k}}{n} \text{ and } \frac{1}{\bar{y}} = \frac{\sum_{k=1}^{n} \frac{1}{y_k}}{n} \qquad (5)$$

Equations (3) to (5) were used to compute the various centroid values needed for interpolation of geoid and orthometric heights.

The other type of average is the central value termed the median which is not considered in this investigation since it is based on different assumptions. For a set of n numbered values arranged from either smallest to largest or vice versa, when n is odd, then the middle value becomes the median. For n is even the mean of a pair of middle values is taken as the median.

Geometric geoid models

Geometric geoid models are determined by finding the differences between the ellipsoidal and the orthometric heights of selected points to obtain the geoid heights of the points.[10] A geometric geoid surface is then fitted to the computed geoid heights of the points to enable geoid heights of new points within the study area be computed. Polynomial surfaces are used to represent continuous surfaces over study areas.[11] The Multiquadratic surface used for the interpolation of geoid heights is given in equation (6).[11]

$$N = a_0 + a_1 X + a_2 Y + a_3 X^2 + a_4 Y^2 + a_5 XY + a_6 X^2 Y + a_7 XY^2 + a_8 X^2 Y^2 \quad \text{- (6)}$$

Where,

$Y = ABS(y - y_o)$

$X = ABS(x - x_o)$

y = Northing coordinate of observed station.

x = Easting coordinate of observed station.

y_o = Northing coordinate of the origin (average of the northing coordinates).

x_o = Easting coordinate of the origin (average of the easting coordinates).

N is geoidal undulation at the point of interest.

$a_0, a_1, a_2, \ldots, a_n$ unknown parameters.

The relationship between the orthometric height (H), ellipsoidal height (h) and the geoid height (N) is given in equation (7).[11]

$$H = h - N \quad (7)$$

Computation of root mean square error

The modelling of orthometric heights using the geometric geoid method requires the computation of the model accuracy with the root mean square error (RMSE) index. To compute the accuracy of the model, the orthometric heights of points from the model that is, the orthometric heights of the points obtained from the differences between the model geoid heights and the ellipsoidal heights of the points are compared with their corresponding existing, as well as spirit levelling orthometric heights to obtain the orthometric height residuals. The orthometric height residuals and the total number of the selected points are used for the computation of the RMSE, as well as the accuracy of the model. The RMSE index for the computation of the model accuracy is given in equation (8).[12]

$$RMSE = \pm\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\delta H_{\text{Residual}}\right)^2} \quad (8)$$

Where,

$\delta H_{\text{Residual}} = H_{\text{KNOWN}} - H_{\text{Model}})$

H_{KNOWN} = Known Orthometric Height

H_{Model} = Model Orthometric Height

n = Number of Points

Figure 1: Map of the Nigerian states and FCT Abuja source: Arcinfo shapefile 2010 (ESRI).

Dataset available

In this study, the following datasets were available: list of coordinates obtained from the survey and mapping department, federal capital development authority (FCDA), Abuja. CSRS-PPP post-processed DGPS coordinates and ellipsoidal heights. Nigeria and federal capital territory (FCT) maps (Figures 1 and 2).

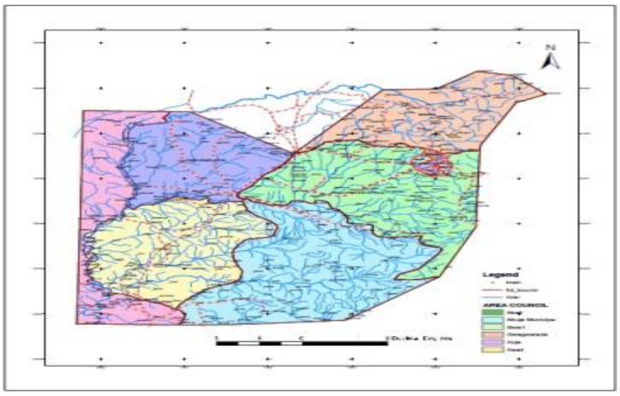

Figure 2: Map of FCT six area councils' source: survey and mapping department, FCDA, Abuja.

METHODS

DGPS coordinates (observations)

Hi-target V30 pro geodetic DGPS dual-frequency receivers were used for data capture in the relative mode of GPS observation. During the observation, each point was occupied for about 2 hours. The observations (RINEX data) were post-processed using CSRS-PPP online software. The results are shown in (Table 1).

Using the post-processing results presented in Table 1, the computations of root mean square centroid was done with equation (4), that of arithmetic mean centroid was done using equation (3) and that of the harmonic mean centroid was carried out with equation (5) as respectively shown in (Table 2 and 3).

The summary, as well as the results of the arithmetic mean, root mean square and the harmonic mean centroids are shown in (Table 4).

A microsoft excel program that was developed using the multiquadratic surface (equation (6)) for interpolation of geoid heights, as well as modelling of orthometric heights within the federal capital territory (FCT), Abuja was used in this study. The computed centroid means were entered accordingly one after the other into the developed program to obtain their respective model orthometric heights as given in (Table 5).

The differences in the respective orthometric heights in Table 5, may arise from the different basis of the development of the theory of each centroid mean type.

The accuracy of the model was computed for each of the calculated centroid means.

The computation was done by comparing the obtained orthometric heights for each centroid mean with the known/spirit levelling orthometric heights using equation (8).

Table 1: Control points coordinates and ellipsoidal heights.

	Coordinate register values			CSRS-PPP
Control point	**Easting (m)**	**Northing (m)**	**Existing orthometric height, H (m)**	**Elip heigth h(m)**
FCC11S	331888.114	998442.043	485.447	509.396
FCT260P	255881.175	993666.807	201.944	224.74
FCT103P	340639.766	998375.578	532.558	556.836
FCT12P	333743.992	1008308.73	735.707	760.192
FCT19P	337452.408	996344.691	635.644	659.824
FCT2168S	310554.927	1009739.93	431.087	455.274
FCT24P	322719.776	1001884.85	453.804	477.987
FCT276P	351983.716	1025998.314	625.572	649.848
FCT4154S	329953.882	1003831.28	476.981	501.232
FCT4159S	326124.422	1003742.86	452.230	476.553
FCT66P	299148.035	998114.283	297.111	321.115
FCT9P	329821.512	1007612.091	497.253	521.693
FCT35P	322183.38	992926.363	427.171	451.299
FCT57P	303234.27	992916.402	323.844	347.795
FCT4028S	330164.634	1001388.24	449.592	473.942
FCT53P	308943.361	993406.773	351.943	375.955
FCT4652S	329441.767	997474.808	462.711	487.113
FCT162P	270791.291	934625.533	189.696	215.091
FCT130P	330982.584	952889.869	695.608	719.383
FCT2327S	282526.612	973821.47	183.287	207.482
FCT2652S	271370.273	945385.429	138.952	163.741
FCT2656S	272644.591	941062.46	204.724	229.237
FCT83P	332954.205	987231.606	568.752	592.801
XP382	284074.729	983364.863	274.586	298.421

Tables 2: Root mean square centroid computation.

Point ID	Easting	SQ Easting	Northing	SQ Northing
FCC11S	331888.114	110149720214.477	998442.043	996886513230.014
FCT260P	255881.175	65475175719.381	993666.807	987373723333.575
FCT103P	340639.766	116035450180.535	998375.578	996753794746.834

Continued.

Point ID	Easting	SQ Easting	Northing	SQ Northing
FCT12P	333743.992	111385052196.096	1008308.73	1016686494994.210
FCT19P	337452.408	113874127664.998	996344.691	992702743283.885
FCT2168S	310554.927	96444362683.975	1009739.93	1019574726236.410
FCT24P	322719.776	104148053821.490	1001884.85	1003773252659.520
FCT276P	351983.716	123892536329.169	1025998.314	1052672540330.840
FCT4154S	329953.882	108869564246.870	1003831.28	1007677238706.440
FCT4159S	326124.422	106357138624.834	1003742.86	1007499729000.980
FCT66P	299148.035	89489546844.361	998114.283	996232121928.604
FCT9P	329821.512	108782229777.966	1007612.091	1015282125929.390
FCT35P	322183.38	103802130348.224	992926.363	985902762340.408
FCT57P	303234.27	91951022502.433	992916.402	985882981360.626
FCT4028S	330164.634	109008685544.354	1001388.24	1002778407210.300
FCT53P	308943.361	95446000305.976	993406.773	986857016642.274
FCT4652S	329441.767	108531877844.082	997474.808	994955992594.637
FCT162P	270791.291	73327923281.447	934625.533	873524886935.534
FCT130P	330982.584	109549470911.317	952889.869	907999102442.837
FCT2327S	282526.612	79821286488.199	973821.47	948328255432.961
FCT2652S	271370.273	73641825068.095	945385.429	893753609365.514
FCT2656S	272644.591	74335073001.557	941062.46	885598553621.252
FCT83P	332954.205	110858502627.182	987231.606	974626243885.339
XP382	284074.729	80698451656.423	983364.863	967006453783.009
Total no. of points=24	Summation of SQ Easting (A) =	2365875207883.440	Summation of SQ Northing (B)=	23500329269995.400
	(A)/24 =	98578133661.810	(B)/24 =	979180386249.808
Computed RMS centroid		313971.5491		989535.4396

Table 3: Arithmetic and harmonic means centroid computation.

	Coordinate register value			CSRS-PPP
Control point	**Easting (m)**	**Northing (m)**	**Existing orthometric height, (m)**	**Elip height (m)**
FCC11S	331888.114	998442.043	485.447	509.396
FCT260P	255881.175	993666.807	201.944	224.74
FCT103P	340639.766	998375.578	532.558	556.836
FCT12P	333743.992	1008308.73	735.707	760.192
FCT19P	337452.408	996344.691	635.644	659.824
FCT2168S	310554.927	1009739.93	431.087	455.274
FCT24P	322719.776	1001884.85	453.804	477.987
FCT276P	351983.716	1025998.314	625.572	649.848
FCT4154S	329953.882	1003831.28	476.981	501.232
FCT4159S	326124.422	1003742.86	452.230	476.553
FCT66P	299148.035	998114.283	297.111	321.115
FCT9P	329821.512	1007612.091	497.253	521.693
FCT35P	322183.38	992926.363	427.171	451.299
FCT57P	303234.27	992916.402	323.844	347.795
FCT4028S	330164.634	1001388.24	449.592	473.942
FCT53P	308943.361	993406.773	351.943	375.955
FCT4652S	329441.767	997474.808	462.711	487.113
FCT162P	270791.291	934625.533	189.696	215.091
FCT130P	330982.584	952889.869	695.608	719.383
FCT2327S	282526.612	973821.47	183.287	207.482
FCT2652S	271370.273	945385.429	138.952	163.741
FCT2656S	272644.591	941062.46	204.724	229.237
FCT83P	332954.205	987231.606	568.752	592.801

Continued.

	Coordinate register value			CSRS-PPP
Control point	**Easting (m)**	**Northing (m)**	**Existing orthometric height, H (m)**	**Elip height h(m)**
XP382	284074.729	983364.863	274.586	298.421
Summation	7509223.422	23742555.27		
Arithmetic mean	312884.3093	989273.1364		
Harmonic mean	310555.900	988733.816		
Total no. of points=24				

Table 4: Arithmetic mean, root mean square, harmonic mean centroid values.

Centroid type	Easting (X_0) m	Northing (Y_0) m	Remarks
Arithmetic mean M_1	312884.309	989273.136	
Root means square mean M_2	313971.549	989535.440	
Harmonic mean M_{-1}	310555.900	988733.816	

Table 5: Orthometric heights obtained for different centroid means.

Point ID	Arithmetic mean centroid H (m)	RMS centroid H (m)	Harmonic mean centroid H (m)
FCC11S	485.161	485.172	485.147
FCT260P	201.963	202.010	201.850
FCT103P	532.681	532.672	532.710
FCT12P	735.826	735.825	735.838
FCT19P	635.703	635.702	635.714
FCT2168S	431.087	431.079	431.099
FCT24P	453.807	453.841	453.743
FCT276P	625.580	625.691	625.287
FCT4154S	476.896	476.910	476.876
FCT4159S	452.269	452.294	452.227
FCT66P	296.925	296.920	296.951
FCT9P	497.334	497.343	497.327
FCT35P	427.252	427.255	427.238
FCT57P	323.747	323.751	323.753
FCT4028S	449.642	449.657	449.619
FCT53P	351.944	351.929	351.993
FCT4652S	462.916	462.930	462.892
FCT162P	189.694	189.125	190.751
FCT130P	695.579	695.522	695.661
FCT2327S	183.221	183.242	183.184
FCT2652S	138.960	138.671	139.485
FCT2656S	204.715	204.366	205.346
FCT83P	568.910	568.880	568.980
XP382	274.399	274.410	274.383

RESULTS

Table 6, presents the accuracy of the model at various centroid means. This was done to show that the various centroid means types used in local geoid, as well as orthometric height modelling, have impacts on the accuracy of the model and to determine which of the centroid mean types is the best for geometric geoid model centroid mean computation.

Table 6: RMSE of different centroid means.

Centroid mean type	Arithmetic mean (m)	Root mean square (m)	Harmonic mean (m)
Orth. height RMSE (m)	0.11	0.187	0.303

DISCUSSION

It can be seen from Table 6, that the computed RMSE, as well as the accuracy of the arithmetic mean, root mean square and the harmonic mean centroids are respectively 0.110m, 0.187m and 0.303m. This implies that the centroid mean type used in geometric geoid, as well as orthometric height modelling, has an impact on the accuracy of the model. The variation in the accuracy of the model was as a result of the different theory which each centroid mean type development was based. It can also be seen from Table 6, that the arithmetic mean centroid has the smallest RMS error, as well as the highest accuracy which also shows that the arithmetic mean centroid is the best for geometric geoid, as well as orthometric height modelling. The centroid mean whose accuracy is the highest gives the most accurate interpolated orthometric height. The various centroid mean types have not really been applied and compared in any previous study.[6-9]

CONCLUSION

In conclusion, the results obtained in this study has shown that the centroid mean type used in local geometric geoid, as well as orthometric height modelling, has an impact on the accuracy of the model. The study has also revealed that the arithmetic mean centroid computation method is the best.

Recommendations

It is recommended that the arithmetic mean method should be adopted for centroid computation, as well as orthometric height modelling using the geometric method.

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