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Research Article

Calculating the MOND constant and addressing flat galactic orbital star rotation velocity curves

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ABSTRACT

A particle based explanation for the MOND constant, a_o, is proposed. For stars orbiting in the outer region of galaxies Newton's inverse square law fails at the same classical angular acceleration v^2/R, (= a_o), regardless of the mass of the galaxy or star, the star orbital velocity v, or the orbit radius R. Gravitational orbital dynamics, where orbital matter propagates un-accelerated in curved space-time as described by General Relativity, applies equally to systems of vastly different masses, i.e. at cosmological and elementary particle scales, with gravity simply an observer domain manifestation of the force localizing electromagnetic energy to form particles. Comparison of astronomical data and particle concepts, with scale adjustments, enables an expression for a_o to be derived and a numerical value obtained within the uncertainty bounds of empirical data. Newton's law fails and stellar orbital velocities become independent of the orbit radius at the same gravitational field strength as at the proton radius. A quantum loop based cause of orbital velocity curve flattening is proposed.

Keywords: MOND constant, Galactic rotation curves, Curved space-time, Gravitational field strength, Newton's law

INTRODUCTION

Cosmological observations of the velocity of stars orbiting the outer region of galaxies where gravity is very weak shows Newton's inverse square law $F_g = G_N m_1 m_2/R^2$, fails at the same classical angular acceleration v^2/R regardless of the mass of the galaxy m_1, or star m_2, the star orbital velocity v, or the orbit radius R. Newton's law indicates the orbital velocity should decrease as $R^{1/2}$, but the 'law' fails at a critical radius R_c, different for every system, as illustrated in Figure 1. At R_c the orbital velocity v becomes independent of the radius R and star rotational velocity curves flatten. The classical angular acceleration value v^2/R at which the 'law' fails is called the MOND constant a_o after Milgrom.[1] There is no accepted explanation for this phenomenon.

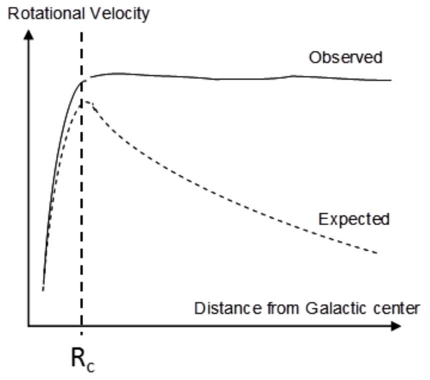

Figure 1: The expected vs. observed rotational velocity of outlying stars as a function of their distance from a galactic center and illustrates the velocity flattening typically observed.

This article discusses the nature of the constant and derives an expression for a_o giving a numerical value within the uncertainty range obtained from empirical data. It also speculates star orbital velocity curve flattening stems from internal proton characteristics related to gravitational quantization.

Newton's inverse square law is based on macroscopic observations using classical mechanics and applies in relatively strong gravitational fields such as on Earth and throughout the Solar system, and is consistent with simple geometric considerations where the gravitational attraction decreases with distance R as $1/R^2$. Similarly, the classical angular acceleration v^2/r applies for a mass rotating on a string where string tension supplies the radial force causing centripetal acceleration in flat observer space-time.

But General Relativity Theory (GRT) shows gravity acts via curved space-time and is not force in observer space. This effectively alters the dimensions of Newton's equation as in [2], and shows orbiting stars propagate un-accelerated in curved space-time and are not subject to radial force. Thus describing the circumstance at which the $1/R^2$ law fails in terms of classical angular acceleration is only an observational convenience and is at best misleading. It is erroneous to equate the gravitational attraction in orbital systems with centripetal acceleration as shown in (1).

$$G_N m_1 m_2/R^2 \neq m_2 v^2/R \qquad (1)$$

Thus while the observational data is valid the reason for the law's failure is not due directly to the orbiting stars classical angular acceleration v^2/R, but to the local gravitational field. As $G_N m_1/R^2$ gives the gravitational field strength at radius R due to the mass m_1, it is evident Newton's law fails not due to angular acceleration, but at the same very low gravitational field strength in each system. This must be true for every orbital mass system orbiting under gravity. It is the field strength which determines the apparent angular acceleration. Hence the MOND phenomenon must occur at the same field strength for all orbital systems wherein mass propagates in curved space-time, regardless of their mass.

Further, as gravity is an emergent property at the elementary particle scale [2], it follows $a_{o,}$ must arise due to a particle related characteristic that causes a significant change in the nature of the gravitational field at a specific very low value. Thus analysis of particle concepts provides a means to evaluate a_o.

OBSERVATIONAL DATA

As a reference point and a means to evaluate cosmological observations, erroneously equating Newton's classical gravitational attraction expression with centripetal force for a mass m_2 orbiting a more massive body at radius r and speed v as in (1), we obtain

$$v^2 = G_N m_2/R \qquad (2)$$

This shows v decreases with radius as $1/R^{1/2}$. But as shown in Figure 2, this relation fails for stars orbiting in the outer regions of galaxies and the orbital velocity of stars becomes constant beyond a critical radius, R_c, where the classical angular acceleration v^2/R (= a_o), has the same value for every system. I.e. a_o identifies an orbital transition point. The figure shows two typical examples obtained from unreferenced data found on the internet.

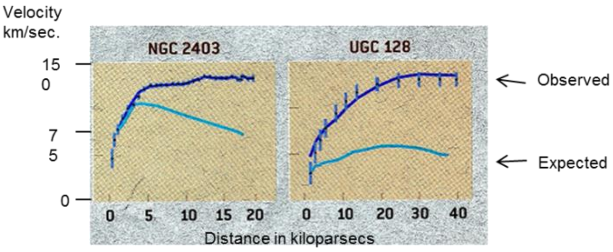

Figure 2: Observed star rotational velocity curves for two Galactic systems.

An empirical value for a_o has been determined by McGaugh [3], via analysis of numerous cosmological observations made by others and his result (3) is detailed in Figure 3 below.

$$a_o = v^2/R_c \sim 1.24 \pm 0.14 \, x \, 10^{-10} m/s^2 \qquad (3)$$

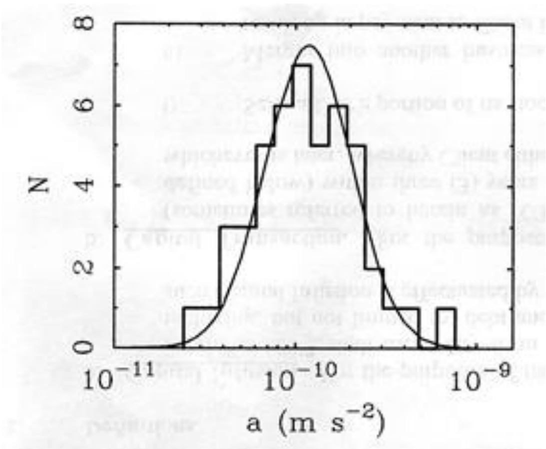

Figure 3: Data for the value of a_o from McGaugh's paper.

DISCUSSION AND ANALYSIS

The following analysis and calculation is based on all masses orbiting in curved space-time obeying the same laws of physics regardless of the scale of the system. This applies to stars orbiting a galaxy, planets orbiting a star, moons orbiting a planet, many particles orbiting a point without a central body, or an electromagnetic (EM) wave exhibiting mass by relativistic propagation in a quantum loop and essentially forming an orbiting matter shell with a radius at the particle scale. In each of these systems the mass energy propagates rectilinearly in curved space-time.

Consider a mass of N protons each of mass m_p and radius r_p where the attraction A due to a gravitational field at a distance d from the mass scales in proportion to Nm_p/d^2 and $d = Xr_p$. Thus for a proportionality constant K,

$$A = KNm_p/(Xr_p)^2 \qquad (4)$$

For a single proton N = 1 and X = 1 at the particle radius, and the effective attraction is $A_p = Km_p/r_p^2$. For a massive body composed of a large number of protons, N >>1, the attracting field will continually decrease with increasing distance as $N/(Xr_p)^2$ until it reaches the same value A_p, as at the proton radius. Any mass orbiting in a gravitational field of this same strength in the far reaches of a galaxy can be considered equivalent to the proton mass circulating at its radius.

A recent analysis [2] of a conceptual model of the electron as an electromagnetic (EM) photon propagating in a relativistic quantum state close to an event horizon provides a theoretical basis for Newton's gravitational equation. The analysis shows his empirical constant G_N, is not fundamental and arose from a simple equation formulated to address and quantify gravity. The electron model consists of a single photon propagating rectilinearly at a relativistic velocity near a toroidal space-time event horizon of closed geometry. By general relativity theory stars orbiting galaxies are also propagating rectilinearly and essentially un-accelerated in space-times curved at the orbit radius due to the mass of the central body, albeit at much lower velocities. I.e. the basic physics are the same although the systems are of vastly different scales and velocities.

The first order expression for G developed in [2] is shown in (5) where α is the fine structure constant ~1/137, m_e the electron rest mass, and ħ is the reduced Planck constant. The numerical value for G obtained by (5) is very close to that empirically determined for G_N, but as gravity acts via curved space-time the derived value differs by c^4 from G_N, previously erroneously assumed to be of dimension $\hbar c/m^2$. Initial measurements of G_N conducted in 1798 by Cavendish using cm.gm.s. (cgs) units led to the inadvertent inclusion of c^4 within G, masquerading as a dimensionless value of $(2.998 \times 10^{10})^4 = 8.0776 \times 10^{41}$, $\underline{c}^4$ say.

$$G \approx \hbar c/(\alpha^{-2/3} m_e \underline{c}^2)^2 \qquad (5)$$

Recognizing the inadvertent inclusion of $\underline{c}^4$ in G_N leads directly to a determination of the MOND constant via a conceptual particle model. With $\hbar c/R^2$ having dimensions of force and inserting the expression for G as in (5) into Newton's equation shows gravitational attraction in observer space has dimensions of force/c^4.

$$\begin{aligned} Gm_1m_2/R^2 &= [\hbar c/(\alpha^{-2/3} m_e \underline{c}^2)^2] m_1m_2/R^2 \\ &= [\hbar c/R^2(\alpha^{-2/3} m_e \underline{c}^2)^2] m_1m_2 \\ &= \text{force}/\underline{c}^4 \qquad (6) \end{aligned}$$

The curved metric orthogonality described in [2] shows far field observer space gravity is ~c^4 weaker than the force constraining EM energy to circulate near the event horizon within an electron. The derived value of G for the electron is the same as empirically determined for all matter, (i.e. G_N), so it is posited, as with the electron, the force constraining the proton energy to its curved metric locality is nominally $\underline{c}^4$ stronger than observer space gravity. From (1), with m_1, m_2 the proton mass m_p, R the proton radius r_p, and adjusting by $\underline{c}^4$ we obtain (7), showing equilibrium between the apparent gravitational attraction and centripetal effect in observer space for a single proton.

$$G_N m_p^2/r_p^2 \sim m_p v^2/\underline{c}^4 r_p \qquad (7)$$

This shows the classical angular acceleration of the proton energy propagating in its curved metric is reduced to an observer space effect of $v^2/\underline{c}^4 r_p$, numerically less than classically assumed by $\underline{c}^4$ and of dimension force/c^4.

This effect can be numerically estimated using the proton radius $r_p = 0.86 \times 10^{-15}$ [4]-[7], a nominal rotation velocity v ~ c (= 2.99792$\times 10^8$m/s), and $\underline{c}^4 = 8.078 \times 10^{41}$, giving a value;

$$c^2/\underline{c}^4 r_p = 1.2937 \times 10^{-10} \qquad (8)$$

However, the energy velocity circulating in the proton cannot be exactly c or by relativity theory the particle would not be evident to the observer. As G is the same for the electron and all matter, as in the electron the proton's energy circulation can be considered toroidal, and distributed about the x, y, and z axes in the ratios $\alpha^{2/3}.\alpha^{2/3}.\alpha^{-1/3}$. If so, the proton energy velocity in its curved space-time is $v = c(1-\alpha^{2/3})^{1/2}$, where $\alpha^{2/3}$ = 0.03762, and v^2/c^2 = 0.96238. With a proton rotational energy velocity $v^2 = 0.96238c^2$ we obtain;

$$v^2/r_p\underline{c}^4 = 1.245 \times 10^{-10} \qquad (9)$$

This numerical result lies within the error bounds of McGaugh's estimate for a_o (= 1.24 ± 0.14 $\times 10^{-10}$m/s^2), although the dimensions obviously differ by c^{-4}.

It is evident the observer space gravitational field strength at the proton energy radius is the same at which Newton's equation fails in galactic systems, i.e. as classically indicated via $v^2/R = a_o$. Larger mass systems are essentially composed of large numbers of protons, which in aggregate increase the gravitational field strength and extend the critical energy radius at which Newton's law fails from r_p to R_c.

The gravitational field of a proton relates to the energy of its constituents which via quantum electrodynamic considerations are considered quantized. At distances beyond the critical radius the gravitational field decreases below that at the proton radius and is sufficiently weak that macroscopic classical gravity is no longer valid and the gravitational field transitions to a realm in which sub particle quantum effects emerge and the inverse square law evidently fails.

It is thereby postulated the origin of the MOND constant lies with the EM energy circulating un-accelerated in the curved space-time localizing the proton energy.

SPECULATION REGARDING ORBITAL VELOCITY CURVE FLATTENING

As in [2], elementary particles can be described in terms of quantum loops wherein the particle energy orbits in one or more planes which rotate about particle axes to form a 3D particle, and where the relativistic state of the loop energy strains the local space-time metric and forms a gravitational field. The metric strain necessarily lies in the direction of energy propagation, i.e. in the plane of the quantum loop and is therefore one dimensional for each loop. Multiple particles create multiple strains with random orientations which can be resolved into two orthogonal directions, so the resulting far field metric strain is areal in nature and gravity decreases with distance as $1/R^2$. But in the circumstance where the gravitational field is very weak and due essentially to a single quantum loop the metric strain is one dimensional and decreases as 1/R. The MOND constant marks the transition from a 2D strain to a 1D strain field.

The classical high field strength angular acceleration is $GM/R^2 = v^2/R$, but if beyond the critical radius R_c at very low gravitational fields the inverse square law reduces to a simple inverse law, the classical angular acceleration will vary as $Gm_1/R = v^2/R$, i.e. the orbital velocity will be independent of R and the orbital rotation velocity curve will be flat as illustrated in Figure 1.

EMPIRICAL VALIDATION

As the transition from a $1/r^2$ to a 1/r gravitational strength law applies at all scales of matter empirical validation may be possible at the laboratory scale. With the proton mass $1.672x10^{-27}$ kg and the radius $0.86x10^{-15}$m, the transition should occur at a distance of about 0.86m for a mass of 10^{30} nucleons, i.e. 1,672kg, e.g. a lead sphere 0.328m radius.

By an incredible coincidence, this is about the same scale of values used for empirical determinations of G_N. Experimenters using a variety of apparatus might therefor obtain slightly different readings for G_N depending on their particular equipment configurations in relation to each MOND transition zone. In the above example the transition zone should extend from 0.86m to about 1.2m, (0.86 + 0.328)m, from the mass center. Evaluating a possible transition zone may be achieved by measuring the attraction change along a radial from a large mass, but establishing absolute values of G_N is not required.

CONCLUSION

The failure of Newton's inverse square law in regions with gravitational field strengths indicated by the MOND constant relates directly to the nature of the proton, specifically the EM energy circulation localized by propagating un-accelerated in a highly curved metric close to an event horizon. The MOND constant identifies the region where the gravitational field strength falls to that at the proton's energy circulation radius, below which other effects emerge and field strength decreases as 1/R.

The gravitational field transition phenomenon should occur at all mass scales so empirical laboratory scale tests may be possible.

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